Chapter 6: Determinants

EXERCISE 6.1 [PAGE 83]

Exercise 6.1 | Q 1.1 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 4 & 7 \\ -7 & 0 \end{vmatrix}$

SOLUTION

 $\begin{vmatrix} 4 & 7 \\ -7 & 0 \end{vmatrix}$

= 4(0) - (-7)(7)= 0 + 49

= 49.

Exercise 6.1 | Q 1.2 | Page 83

	3	$^{-5}$	2
Evaluate the following determinants:	1	8	9
	3	7	0

SOLUTION

$$\begin{vmatrix} 3 & -5 & 2 \\ 1 & 8 & 9 \\ 3 & 7 & 0 \end{vmatrix}$$

= $3\begin{vmatrix} 8 & 9 \\ 7 & 0 \end{vmatrix} - (-5)\begin{vmatrix} 1 & 9 \\ 3 & 0 \end{vmatrix} + 2\begin{vmatrix} 1 & 8 \\ 3 & 7 \end{vmatrix}$
= $3(0 - 63) + 5(0 - 27) + 2(7 - 24)$
= $3(-63) + 5(-27) + 2(7 - 24)$
= $3(-63) + 5(-27) + 2(-17)$
= $-189 - 135 - 34$
= -358 .

Exercise 6.1 | Q 1.3 | Page 83

	1	i	3
Evaluate the following determinants:	i^3	2	5
	3	2	$\mathbf{i^4}$

SOLUTION

$$\begin{vmatrix} 1 & i & 3 \\ i^{3} & 2 & 5 \\ 3 & 2 & i^{4} \end{vmatrix}$$

=
$$\begin{vmatrix} 1 & i & 3 \\ -i & 2 & 5 \\ 3 & 2 & 1 \end{vmatrix}$$
 ...[:: $i^{2} = -1, i^{4} = 1$]
= $1\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix} - i\begin{vmatrix} -i & 5 \\ 3 & 1 \end{vmatrix} + 3\begin{vmatrix} -i & 2 \\ 3 & 2 \end{vmatrix}$
= $1(2 - 10) - i(-i - 15) + 3(-2i - 6)$
= $-8 + i^{2} + 15i - 6i - 18$
= $i^{2} - 26 + 9i$
= $-1 - 26 + 9i$...[:: $i^{2} = -1$]
= $-27 + 9i$.

Exercise 6.1 | Q 1.4 | Page 83

	5	5	5
Evaluate the following determinants:	5	4	4
	5	4	8

SOLUTION

$$= 5 \begin{vmatrix} 4 & 4 \\ 4 & 8 \end{vmatrix} - 5 \begin{vmatrix} 5 & 4 \\ 5 & 8 \end{vmatrix} + 5 \begin{vmatrix} 5 & 4 \\ 5 & 4 \end{vmatrix}$$
$$= 5(32 - 16) -5(40 - 20) + 5(20 - 20)$$
$$= 5(16) - 5(20) + 5(0)$$
$$= 80 - 100$$
$$= -20.$$

Exercise 6.1 | Q 1.5 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 2i & 3 \\ 4 & -i \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} 2\mathbf{i} & 3 \\ 4 & -\mathbf{i} \end{vmatrix}$$

= 2i(-i) - 3(4)
= -2i² - 12
= -2(-1) -12 ...[: i² = -1]
= 2 - 12
= -10.

Exercise 6.1 | Q 1.6 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

= $3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - (-4)\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$

$$= 3(1 + 6) + 4(1 + 4) + 5(3 - 2)$$

= 3(7) + 4(5) + 5(1)
= 21 + 20 + 5
= 46.

Exercise 6.1 | Q 1.7 | Page 83

Evaluate the following determinants:

ahg hbf gfC

SOLUTION

 $\begin{vmatrix} \mathbf{a} & \mathbf{h} & \mathbf{g} \\ \mathbf{h} & \mathbf{b} & \mathbf{f} \\ \mathbf{g} & \mathbf{f} & \mathbf{C} \end{vmatrix}$ $= \mathbf{a} \begin{vmatrix} \mathbf{b} & \mathbf{f} \\ \mathbf{f} & \mathbf{c} \end{vmatrix} - \mathbf{h} \begin{vmatrix} \mathbf{h} & \mathbf{f} \\ \mathbf{h} & \mathbf{c} \end{vmatrix} + \mathbf{g} \begin{vmatrix} \mathbf{h} & \mathbf{b} \\ \mathbf{g} & \mathbf{f} \end{vmatrix}$ $= \mathbf{a}(\mathbf{b}\mathbf{c} - \mathbf{f}^2) - \mathbf{h}(\mathbf{h}\mathbf{c} - \mathbf{g}\mathbf{f}) + \mathbf{g}(\mathbf{h}\mathbf{f} - \mathbf{g}\mathbf{b})$ $= \mathbf{a}\mathbf{b}\mathbf{c} - \mathbf{a}\mathbf{f}^2 - \mathbf{h}^2\mathbf{c} + \mathbf{f}\mathbf{g}\mathbf{h} + \mathbf{f}\mathbf{g}\mathbf{h} - \mathbf{g}^2\mathbf{b}$ $= \mathbf{a}\mathbf{b}\mathbf{c} + 2\mathbf{f}\mathbf{g}\mathbf{h} - \mathbf{a}\mathbf{f}^2 - \mathbf{b}\mathbf{g}^2 - \mathbf{c}\mathbf{h}^2.$ Exercise 6.1 | Q 1.8 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

= $0 \begin{vmatrix} 0 & -c \\ c & 0 \end{vmatrix} - a \begin{vmatrix} -a & -c \\ b & 0 \end{vmatrix} - b \begin{vmatrix} -a & 0 \\ b & c \end{vmatrix}$

$$= 0 - a (0 + bc) - b(-ac - 0)$$

= - a(bc) - b (- ac)
= - abc + abc
= 0.
Exercise 6.1 | Q 2.1 | Page 83

Find the value(s) of x, if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$
$$\therefore 10 - 12 = 5x - 6x$$
$$\therefore -2 = -x$$
$$\therefore x = 2.$$

Exercise 6.1 | Q 2.2 | Page 83

Find the value(s) of x, if $\begin{vmatrix} 2 & 1 & x+1 \\ -1 & 3 & -4 \\ 0 & -5 & 3 \end{vmatrix} = 0$

SOLUTION

$$\begin{vmatrix} 2 & 1 & x+1 \\ -1 & 3 & -4 \\ 0 & -5 & 3 \end{vmatrix} = 0$$

$$\therefore 2\begin{vmatrix} 3 & -4 \\ -5 & 3 \end{vmatrix} - 1\begin{vmatrix} -1 & -4 \\ 0 & 3 \end{vmatrix} + (x+1)\begin{vmatrix} -1 & 3 \\ 0 & -5 \end{vmatrix} = 0$$

$$\therefore 2(9-20) - 1(-3-0) + (x+1)(5-0) = 0$$

$$\therefore 2(-11) - 1(-3) + (x+1)(5) = 0$$

$$\therefore -22 + 3 + 5x + 5 = 0$$

$$\therefore 5x = 14$$
$$\therefore x = \frac{14}{5}$$

Exercise 6.1 | Q 2.3 | Page 83

Evaluate the following determinants: $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$

SOLUTION

$$\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$$

$$\therefore (x-1) \begin{vmatrix} x-2 & x-3 \\ 0 & x-3 \end{vmatrix} - x \begin{vmatrix} 0 & x-3 \\ 0 & x-3 \end{vmatrix} + (x-2) \begin{vmatrix} 0 & x-2 \\ 0 & 0 \end{vmatrix} = 0$$

$$\therefore (x-1)[(x-2)(x-3)-0] - x(0-0) + (x-2)(0-0) = 0$$

$$\therefore (x-1)(x-2)(x-) = 0$$

$$\therefore x-1 = 0 \text{ or } x-2 = 0 \text{ or } x-3 = 0$$

$$\therefore x-1 \text{ or } x = 2 \text{ or } x = 3.$$

Exercise 6.1 | Q 3.1 | Page 83

Solve the following equation : $\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$



SOLUTION

 $egin{array}{c|cccc} x & 2 & 2 \ 2 & x & 2 \ 2 & 2 & x \ \end{array} = 0$ $\therefore x \begin{vmatrix} x & 2 \\ 2 & x \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 2 & x \end{vmatrix} + 2 \begin{vmatrix} 2 & x \\ 2 & 2 \end{vmatrix} = 0$ $\therefore x(x^2 - 4) - 2(2x - 4) + 2(4 - 2x) = 0$ $\therefore x(x^2 - 4) - 2(2x - 4) - 2(2x - 4) = 0$ $\therefore x(x + 2)(x - 2) - 8(x - 2) = 0$ $\therefore x(x + 2)(x - 2) - 8(x - 2) = 0$ $\therefore (x-2)[x(x+2)-8] = 0$ $\therefore (x-2)(x^2+2x-8)=0$ $\therefore (x-2)(x^2 + 4x - 2x - 8) = 0$ $\therefore (x-2)(x+4)(x-2) = 0$ $\therefore (x-2)^2(x+4) = 0$ $\therefore (x-2)^2 = 0 \text{ or } x + 4 = 0$ $\therefore x - 2 = 0 \text{ or } x = -4$ $\therefore x = 2 \text{ or } x = -4.$ Exercise 6.1 | Q 3.2 | Page 83

Solve the following equation : $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

SOLUTION

 $egin{array}{c|cccc} 1 & 4 & 20 \ 1 & -2 & 5 \ 1 & 2x & 5x^2 \ \end{array} = 0$



$$\therefore 1 \begin{vmatrix} -2 & 5 \\ 2x & 5x^2 \end{vmatrix} - 4 \begin{vmatrix} 1 & 5 \\ 1 & 5x^2 \end{vmatrix} + 20 \begin{vmatrix} 1 & -2 \\ 1 & 2x \end{vmatrix} = 0$$

$$\therefore 1(-10x^2 - 10x) - 4(5x^2 - 5) + 20(2x + 2) = 0$$

$$\therefore -10x^2 - 10x - 20^2 + 20 + 40x + 40 = 0$$

$$\therefore -30x^2 + 30x + 60 = 0$$

$$\therefore x^2 - x - 2 = 0 \quad \dots [Dividing throughout by (-30)]$$

$$\therefore x^2 - 2x + x - 2 = 0$$

$$\therefore (x - 2)(x + 1) = 0$$

$$\therefore x - 2 = 0 \text{ or } x + 1 = 0$$

$$\therefore x = 2 \text{ or } x = -1.$$

Exercise 6.1 | Q 4 | Page 83

Find the value of x, if $\begin{vmatrix} x & -1 & 2 \\ 2x & 1 & -3 \\ 3 & -4 & 5 \end{vmatrix} = 29$

SOLUTION

$$\begin{vmatrix} x & -1 & 2 \\ 2x & 1 & -3 \\ 3 & -4 & 5 \end{vmatrix} = 29$$

$$\therefore x \begin{vmatrix} 1 & -3 \\ -4 & 5 \end{vmatrix} - (-1) \begin{vmatrix} 2x & -3 \\ 3 & 5 \end{vmatrix} + 2 \begin{vmatrix} 2x & 1 \\ 3 & -4 \end{vmatrix} = 29$$

$$\therefore x(5 - 12) + 1(10x + 9) + 2(-8x - 3) = 29$$

$$\therefore -7x + 10x + 9 - 16x - 6 = 29$$

$$\therefore -13x + 3 = 29$$

$$\therefore -13x = 26$$

$$\therefore x = -2.$$

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Exercise 6.1 | Q 5 | Page 83 Find x and y if $\begin{vmatrix} 4i & i^3 & 2i \\ 1 & 3i^2 & 4 \\ 5 & -3 & i \end{vmatrix} = x + iy$, where $i = \sqrt{-1}$. SOLUTION $\begin{array}{cccc} {}^{_{\mathbf{4}\mathbf{1}}} & {\rm i}^{_{\mathbf{3}}} & 2{\rm i} \\ 1 & 3{\rm i}^2 & 4 \\ 5 & -3 & {\rm i} \end{array}$ $= \begin{vmatrix} 4i & -i & 2i \\ 1 & -3 & 4 \\ 5 & -3 & i \end{vmatrix} \qquad \dots [\because i^2 = -1]$ $= 4i \begin{vmatrix} -3 & 4 \\ -3 & i \end{vmatrix} - (-i) \begin{vmatrix} 1 & 4 \\ 5 & i \end{vmatrix} + 2i \begin{vmatrix} 1 & -3 \\ 5 & -3 \end{vmatrix}$ = 4i(-3i + 12) + i(i - 20) + 2i(-3 + 15) $= -12i^{2} + 48i + i^{2} - 20i + 24i$ $= -11i^{2} + 52i$ = -11(-1) + 52i ...[: $i^2 = -1$] = 11 + 52i Comparing with x + iy, we get x = 11, y = 52.

EXERCISE 6.2 [PAGE 89]

Exercise 6.2 | Q 1.1 | Page 89 Without expanding evaluate the following determinants : $\begin{vmatrix} 1 & a & b + c \\ 1 & b & c + a \\ 1 & c & a + b \end{vmatrix}$





SOLUTION

Let $D = \begin{vmatrix} 1 & a & b + c \\ 1 & b & c + a \\ 1 & c & a + b \end{vmatrix}$ Applying $C_3 \rightarrow C_3 + C_2$, we get $D = \begin{vmatrix} 1 & a & a + b + c \\ 1 & b & a + b + c \\ 1 & c & a + b + c \end{vmatrix}$ Taking (a + b + c) common from C_3 , we get $D = (a + b + c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$ $\therefore D = (a + b + c)(0) \quad ...[\because C_1 \text{ and } C_3 \text{ are identical}]$ $\therefore D = 0.$ Exercise 6.2 | Q 1.2 | Page 89

Without expanding evaluate the following determinants : $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

SOLUTION

Let D =
$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

Taking (3x) common from R₃, we get

$$D = 3x \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$



= 0.

Without expanding evaluate the following determinants : $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

SOLUTION

	2	7	65
Let D =	3	8	75
	5	9	86

Applying $C_3 \rightarrow C_3 - 9C_2$, we get

$$D = \begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix}$$

= 0 ...[\because C₁ and C₃ are identical]

Exercise 6.2 | Q 2 | Page 89

Using properties of determinants, show that $\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$ = 4abc.

SOLUTION

L.H.S. =
$$\begin{vmatrix} \mathbf{a} + \mathbf{b} & \mathbf{a} & \mathbf{b} \\ \mathbf{a} & \mathbf{a} + \mathbf{c} & \mathbf{c} \\ \mathbf{b} & \mathbf{c} & \mathbf{b} + \mathbf{c} \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - (C_2 + C_3)$, we get
L.H.S. =
$$\begin{vmatrix} \mathbf{0} & \mathbf{a} & \mathbf{b} \\ -2\mathbf{c} & \mathbf{a} + \mathbf{c} & \mathbf{c} \\ -3\mathbf{c} & \mathbf{c} & \mathbf{b} + \mathbf{c} \end{vmatrix}$$

Taking (- 2) common from C1, we get

L.H.S. = $-2\begin{vmatrix} 0 & a & b \\ c & a+c & c \\ c & c & b+c \end{vmatrix}$ Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_2 - C_1$, we get $L.H.S. = -2 \begin{vmatrix} 0 & a & b \\ c & a & 0 \\ c & 0 & b \end{vmatrix}$ = 2[0(ab - 0) - a(bc - 0) + b(0 - ac)]= -2(0 - abc - abc)= -2(-2abc)= 4abc= R.H.S. Exercise 6.2 | Q 3 | Page 89 Solve the following equation: $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$ SOLUTION $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_2 \rightarrow R_3 - R_3$, we get

$$\begin{vmatrix} x+2 & x+6 & x-1 \\ 4 & -7 & 3 \\ -3 & -4 & 7 \end{vmatrix} = 0$$

$$\therefore (x+2)(-49+12) - (x+6)(28+9) + (x-1)(-16-21) = 0$$

$$\therefore (x+2)(-37) - (x+6)(37) + (x-1)(-37) = 0$$



$$\therefore 3x + 7 = 0$$

$$\therefore x = \frac{-7}{3}.$$

Exercise 6.2 | Q 4 | Page 89
If $\begin{vmatrix} 4+x & 4-x & 4-x \\ 4-x & 4+x & 4-x \\ 4-x & 4-x & 4+x \end{vmatrix} = 0$, then find the values of x.

SOLUTION

$$\begin{vmatrix} 4+x & 4-x & 4-x \\ 4-x & 4+x & 4-x \\ 4-x & 4-x & 4+x \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 12 - x & 4 - x & 4 - x \\ 12 - x & 4 + x & 4 - x \\ 12 - x & 4 - x & 4 + x \end{vmatrix} = 0$$

Taking (12 - x) common from C₁, we get

$$(12-x)\begin{vmatrix} 1 & 4-x & 4-x \\ 1 & 4+x & 4-x \\ 1 & 4-x & 4+x \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

$$(12-x)\begin{vmatrix} 1 & 4-x & 4-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

$$\therefore (12-x)[1(4x^2-0) - (4-x)(0-0) + (4-x)(0-0)] = 0$$



$$\therefore (12 - x)(4x^{2}) = 0$$

$$\therefore x^{2}(12 - x) = 0$$

$$\therefore x = 0 \text{ or } 12 - x = 0$$

$$\therefore x = 0 \text{ or } x = 12.$$

Exercise 6.2 | Q 5 | Page 89

	1	3	6		2	3	3		1	2	1
Without expanding determinants, show that	6	1	4	+	2	1	2	= 10	3	1	7
	3	7	12		1	7	6		3	2	6

SOLUTION

L.H.S. =
$$\begin{vmatrix} 1 & 3 & 6 \\ 6 & 1 & 4 \\ 3 & 7 & 12 \end{vmatrix} + 4 \begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix}$$

= $\begin{vmatrix} 2 & 3 & 3 \\ 12 & 1 & 2 \\ 6 & 7 & 6 \end{vmatrix} + \begin{vmatrix} 8 & 3 & 3 \\ 8 & 1 & 2 \\ 4 & 7 & 6 \end{vmatrix}$
= $\begin{vmatrix} 2+8 & 3 & 3 \\ 12+8 & 1 & 2 \\ 6+4 & 7 & 6 \end{vmatrix}$
= $\begin{vmatrix} 10 & 3 & 3 \\ 20 & 1 & 2 \\ 10 & 7 & 6 \end{vmatrix}$

Interchanging rows and columns, we get

L.H.S. =
$$\begin{vmatrix} 10 & 20 & 10 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix}$$

Taking 10 common from R₁, we get

L.H.S. =
$$10\begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix}$$

= R.H.S.
Exercise 6.2 | Q 6.1 | Page 89

 Without expanding determinants, find the value of
 10
 57
 107

 12
 64
 124

 15
 78
 153

SOLUTION

	10	57	107
Let D =	12	64	124
	15	78	153

Applying $C_3 \rightarrow C_3 - C_2$, we get

$$D = \begin{vmatrix} 10 & 57 & 50 \\ 12 & 64 & 60 \\ 15 & 78 & 75 \end{vmatrix}$$

Taking (5) common from C_3 , we get

$$D = 5 \begin{vmatrix} 10 & 57 & 10 \\ 12 & 64 & 12 \\ 15 & 78 & 15 \end{vmatrix}$$

= 5(0) ...[:: C₁ and C₃ are identical]
= 0.
Exercise 6.2 | Q 6.2 | Page 89
Without expanding determinants, find the value of
$$\begin{vmatrix} 2014 & 2017 & 1 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 1 \end{vmatrix}$$

SOLUTION



Let D =
$$\begin{vmatrix} 2014 & 2017 & 1 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 1 \end{vmatrix}$$

Applying C₂ \rightarrow C₂ $-$ C₁, we get
D = $\begin{vmatrix} 2014 & 3 & 1 \\ 2020 & 3 & 1 \\ 2023 & 3 & 1 \end{vmatrix}$
Taking (3) common from C₂, we get
D = $3 \begin{vmatrix} 2014 & 1 & 1 \\ 2020 & 1 & 1 \\ 2023 & 1 & 1 \end{vmatrix}$
= 3(0) ...[\because C₂ and C₃ are identical]
= 0.
Exercise 6.2 | Q7.1 | Page 89

Exercise 0.2 Q 1.1 Page 09										
	\mathbf{a}_1	$\mathbf{b_1}$	c_1	$\mathbf{b_1}$	$\mathbf{c_1}$	\mathbf{a}_1	$\mathbf{c_1}$	\mathbf{a}_1	$\mathbf{b_1}$	
Without expanding determinants, prove that	\mathbf{a}_2	\mathbf{b}_2	$ \mathbf{c}_2 =$	\mathbf{b}_2	\mathbf{c}_2	\mathbf{a}_2	$= \mathbf{c}_2$	\mathbf{a}_2	\mathbf{b}_2	
	\mathbf{a}_3	\mathbf{b}_3	c ₃	\mathbf{b}_3	\mathbf{c}_3	\mathbf{a}_3	\mathbf{c}_3	\mathbf{a}_3	\mathbf{b}_3	

SOLUTION

Let D =
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$
 ...(i)
Let E = $\begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix}$

Applying $C_1 \leftrightarrow C_2$, we get

$$\mathsf{E} = - \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$$

$$E = - \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$$

Applying $C_1 \leftrightarrow C_3$, we get

$$E = -(-1) \begin{vmatrix} a_{1} & b_{1} & c_{1} \\ a_{2} & b_{2} & c_{2} \\ a_{3} & b_{3} & c_{3} \end{vmatrix}$$

$$\therefore E = \begin{vmatrix} a_{1} & b_{1} & c_{1} \\ a_{2} & b_{2} & c_{2} \\ a_{3} & b_{3} & c_{3} \end{vmatrix}$$
...(ii)
$$Let F = \begin{vmatrix} c_{1} & a_{1} & b_{1} \\ c_{2} & a_{2} & b_{2} \\ c_{3} & a_{3} & b_{3} \end{vmatrix}$$

Applying $C_{1} \leftrightarrow C_{2}$, we get
$$F = - \begin{vmatrix} a_{1} & c_{1} & b_{1} \\ a_{2} & c_{2} & b_{2} \\ a_{3} & c_{3} & b_{3} \end{vmatrix}$$

Applying $C_{2} \leftrightarrow C_{3}$, we get
$$F = -(-1) \begin{vmatrix} a_{1} & b_{1} & c_{1} \\ a_{2} & b_{2} & c_{2} \\ a_{3} & b_{3} & c_{3} \end{vmatrix}$$

...(iii)



From (i), (ii) and (iii), we get

 $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$

Exercise 6.2 | Q 7.2 | Page 89

	1	yz	y+z	1	x	x^2
Without expanding determinants, prove that	1	zx	z+x =	1	\boldsymbol{y}	y^2
	1	xy	x+y	1	z	z^2

SOLUTION

L.H.S. =
$$\begin{vmatrix} 1 & yz & y+z \\ 1 & zx & z+x \\ 1 & xy & x+y \end{vmatrix}$$

= $\begin{vmatrix} 1 & yz & x+y+z-x \\ 1 & zx & y+z+x-y \\ 1 & xy & z+x+y-z \end{vmatrix}$
= $\begin{vmatrix} 1 & yz & x+y+z \\ 1 & zx & x+y+z \\ 1 & zx & x+y+z \end{vmatrix} + \begin{vmatrix} 1 & yz & -x \\ 1 & zx & -y \\ 1 & xy & x+y+z \end{vmatrix}$
= $\begin{vmatrix} 1 & yz & x+y+z \\ 1 & zx & -y \\ 1 & xy & x+y+z \end{vmatrix} + \begin{vmatrix} 1 & yz & -x \\ 1 & zx & -y \\ 1 & zy & -z \end{vmatrix}$

In 1st determinant, taking (x + y + z) common from C₃ and in 2nd determinant, taking $\frac{1}{x}$, $\frac{1}{y}$, $\frac{1}{z}$ common from R₁, R₂, R₃ respectively, we get



L.H.S. =
$$(x + y + z) \begin{vmatrix} 1 & yz & 1 \\ 1 & zx & 1 \\ 1 & xy & 1 \end{vmatrix} - \frac{1}{xyz} \begin{vmatrix} x & xyz & x^2 \\ y & xyz & y^2 \\ z & xyz & z^2 \end{vmatrix}$$

In 2^{nd} determinant, taking xyz common from C_{2} , we get

L.H.S. =
$$(x + y + z)(0) - \frac{xyz}{xyz} \begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix}$$
 ...[:: C₁ and C₂ are identical in 1st determinant]
= $-\begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix}$
Applying C₁ \leftrightarrow C₂, we get

L.H.S. =
$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$
 = R.H.S.

EXERCISE 6.3 [PAGES 93 - 94]

Exercise 6.3 | Q 1.1 | Page 93

Solve the following equations using Cramer's Rule: x + 2y - z = 5, 2x - y + z = 1, 3x + 3y = 8

SOLUTION

Given equations are x + 2y - z = 5 2x - y + z = 1 3x + 3y = 8i.e. 3x + 3y + 0z = 8 $1 \quad 2 \quad -1$

$$\therefore D = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 3 & 0 \end{vmatrix}$$

= 1(0 - 3) - 2(0 - 3) - 1(6 + 3)
= -3 + 6 - 9
= -6

$$D_{X} = \begin{vmatrix} 5 & 2 & -1 \\ 1 & -1 & 1 \\ 8 & 3 & 0 \end{vmatrix}$$

$$= 5(0-3) - 2(0-8) + (-1)(3+8)$$

$$= -15 + 16 - 11$$

$$= -10$$

$$D_{y} = \begin{vmatrix} 1 & 5 & -1 \\ 2 & 1 & 1 \\ 3 & 8 & 0 \end{vmatrix}$$

$$= 1(0-8) - 5(0-3) + 1(6-3)$$

$$= -8 + 15 - 13$$

$$= -6$$

$$D_{z} = \begin{vmatrix} 1 & 2 & 5 \\ 2 & -1 & 1 \\ 3 & 3 & 8 \end{vmatrix}$$

$$= 1(-8-3) - 2(16-3) + 5(6+3)$$

$$= -11 - 26 + 45$$

$$= 8$$
By Cramer's Rule,
$$x = \frac{D_{y}}{D} = \frac{-10}{-6} = \frac{5}{3}$$

$$y = \frac{D_{y}}{D} = \frac{-6}{-6} = 1$$

$$z = \frac{D_{z}}{D} = \frac{8}{-6} = \frac{-4}{3}$$

$$\therefore x = \frac{5}{3}, y = 1 \text{ and } z = \frac{-4}{3} \text{ are the solution of the given equations.}$$
Exercise 6.3 | Q 12 | Page 93
Solve the following equations using Cramer's Rule: 2x - y + 6z = 10, 3x + 4y - 5z = 11.

8x - 7y - 9z = 12





SOLUTION

Given equations are

2x - y + 6z = 103x + 4y - 5z = 118x - 7y - 9z = 12 $\mathsf{D} = \begin{vmatrix} 2 & -1 & 6 \\ 3 & 4 & -5 \\ 8 & -7 & -9 \end{vmatrix}$ = 2(-36-35) - (-1)(-27 + 40) + 6(-21 - 32) = - 142 + 13 - 318 = - 447 $\mathsf{D}_{\mathsf{X}} = \begin{vmatrix} 10 & -1 & 6 \\ 11 & 4 & -5 \\ 12 & -7 & -9 \end{vmatrix}$ = 10(-36 - 35) - (-1)(-99 + 60) + 6(-77 - 48)= -710 - 39 - 750 = - 1499 $\mathsf{D}_{\mathsf{y}} = \begin{vmatrix} 2 & 10 & 6 \\ 3 & 11 & -5 \\ 8 & 12 & -9 \end{vmatrix}$ = 2(-99 + 60) - 10(-27 + 40) + 6(36 - 88)= -78 - 130 - 312= - 520 $\mathsf{D}_{\mathsf{X}} = \begin{vmatrix} 2 & -1 & 10 \\ 3 & 4 & 11 \\ 8 & -7 & 12 \end{vmatrix}$



By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-1499}{-447} = \frac{1499}{447}$$

$$y = \frac{D_y}{D} = \frac{-520}{-447} = \frac{520}{447}$$

$$z = \frac{D_z}{D} = \frac{-332}{-447} = \frac{332}{447}$$

$$\therefore x = \frac{1449}{447}, y = \frac{520}{447} \text{ and } z = \frac{332}{447} \text{ are the solutions of the given equations.}$$

Exercise 6.3 | Q 1.4 | Page 93

Solve the following equations using Cramer's Rule:

 $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -2, \ \frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 3, \ \frac{2}{x} - \frac{1}{y} + \frac{3}{z} = -1$

SOLUTION

Let
$$rac{1}{x}$$
 p, $rac{1}{y}$ = q, $rac{1}{z}$ = r

.: The given equations become

$$p + q + r = -2$$

$$p - 2q + r = 3$$

$$2p - q + 3r = -1$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$= 1(-6 + 1) - 1(3 - 2) + 1(-1 + 4)$$

$$= 5 - 1 + 3$$

$$= -3$$

$$D_{p} = \begin{vmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & -1 & 3 \end{vmatrix}$$

$$= -2(-6+1) - 1(9+1) + 1(-3-2)$$

$$= 10 - 10 - 5$$

$$= -5$$

$$D_{q} = \begin{vmatrix} 1 & -2 & 1 \\ 1 & 3 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$= 1(9+1) + 2(3-2) + 1(-1-6)$$

$$= 10 + 2 - 7$$

$$= 5$$

$$D_{r} = \begin{vmatrix} 1 & 1 & -2 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}$$

$$= 1(2+3) - 1(-1-6) - 2(-1+4)$$

$$= 5+7-6$$

$$= 6$$
By Cramer's Rule,
$$p = \frac{D_{p}}{D} = \frac{-5}{-3} = \frac{5}{3}$$

$$q = D - \frac{q}{q} = \frac{-5}{-3},$$

$$r = \frac{D_{r}}{D} = \frac{6}{-3} = -2$$

$$\therefore \frac{1}{x} = \frac{5}{3}, \frac{1}{y} = \frac{-5}{3}, \frac{1}{z} = -2$$



 \therefore x = $\frac{3}{5}$, y = $\frac{-3}{5}$, z = $\frac{-1}{2}$ are the solution of the given equatios.

Exercise 6.3 | Q 1.5 | Page 93

Solve the following equations using Cramer's Rule:

 $\frac{2}{x} - \frac{1}{y} + \frac{3}{z} = 4, \frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 2, \frac{3}{x} + \frac{1}{y} - \frac{1}{z} = 2$

SOLUTION

Let
$$\frac{1}{x} = p, \frac{1}{y} = q, \frac{1}{z} = r$$

 \therefore The given equations become
 $2p - q + 3r = 4$
 $p - q - r = 2$
 $3p + q - r = 2$
 $D = \begin{vmatrix} 2 & -1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -1 \end{vmatrix}$
 $= 2(1 - 1) - (-1)(-1 - 3) + 3(1 + 3)$
 $= 0 - 4 + 12$
 $= 8$
 $D_p = \begin{vmatrix} 4 & -1 & 3 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$
 $= 4(1 - 1) - (-1)(-2 - 2) + 3(2 + 2)$
 $= 0 - 4 + 12$
 $= 8$



$$D_{q} = \begin{vmatrix} 2 & 4 & 3 \\ 1 & 2 & 1 \\ 3 & 2 & -1 \end{vmatrix}$$

= 2(-2 - 2) -4(-1 - 3) + 3(2 - 6)
= -8 + 16 - 12
= -4
$$D_{r} = \begin{vmatrix} 2 & -1 & 4 \\ 1 & -1 & 2 \\ 3 & 1 & 2 \end{vmatrix}$$

= 2(-2 - 2) - (-1)(2 - 6) + 4(1 + 3)
= -8 - 4 + 16
= 4

By Cramer's Rule,

p =	$\frac{D_p}{D} =$	$=\frac{8}{8}=$	1		
q =	$\frac{D_q}{D} =$	$=\frac{-4}{8}$	= -	$\frac{-1}{2}$	
r = -	$\frac{D_r}{D} =$	$\frac{4}{8} =$	$\frac{1}{2}$		
$\frac{1}{x}$	$\frac{1}{2} = 1,$	$rac{1}{y} =$	$\frac{-1}{2}$	$\frac{1}{z} =$	= 1/2

 \therefore x = 1, y = -2 and z = 2 are the solutions of the given equations.

Exercise 6.3 | Q 2 | Page 93

An amount of ₹ 5,000 is invested in three plans at rates 6%, 7% and 8% per annum respectively. The total annual income from these investments is ₹ 350. If the total annual income from first two investments is ₹ 70 more than the income from the third, find the amount invested in each plan by using Cramer's Rule.

CLICK HERE

SOLUTION

Let the amount of each investment be ₹ x, ₹ y and ₹ z. According to the given conditions,

x + y + z = 50006%x + 7y + 8z = 350

$$\therefore \frac{6}{100}x + \frac{7}{100}y + \frac{8}{100}z = 350$$

$$\therefore 6x + 7y + 8z = 35000$$

$$6\%x + 7\%y = 8\%z + 70$$

$$\therefore \frac{6}{100}x + \frac{7}{100}y = \frac{8}{100}z + 70$$

$$\therefore 6x + 7y = 8z + 7000$$

$$\therefore 6x + 7y - 8z = 7000$$

$$\therefore 6x + 7y - 8z = 7000$$

$$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix}$$

$$= 1(-56 - 56) - 19 - 48 - 48) + 1(42 - 42)$$

$$= -112 + 96 + 0$$

$$= -16$$

$$D_x = \begin{vmatrix} 5000 & 1 & 1 \\ 35000 & 7 & 8 \\ 7000 & 7 & -8 \end{vmatrix}$$

Taking 1000 common from C_1 , we get

$$D_{\rm X} = \begin{vmatrix} 5 & 1 & 1 \\ 35 & 7 & 8 \\ 7 & 7 & -8 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - {}^5C_3$ and $C_2 \rightarrow C_2 - C_3$, we get

$$D_{\rm X} = 1000 \begin{vmatrix} 0 & 0 & 1 \\ -5 & -1 & 8 \\ 47 & 15 & -8 \end{vmatrix}$$

$$= 1000 [0 - 0 + 1(-75 + 47)]$$
$$= 1000 \times (-28) = -28000$$
$$D_{y} = \begin{vmatrix} 1 & 5000 & 1 \\ 6 & 35000 & 8 \\ 6 & 7000 & -8 \end{vmatrix}$$

Taking 1000 common from C_2 , we get

Taking 1000 common from C_3 , we get

$$\begin{array}{c|c} \mathsf{D}_{z} = \mathbf{1000} \begin{vmatrix} 0 & 1 & 5 \\ 6 & 7 & 35 \\ 6 & 7 & 7 \end{vmatrix}$$

Applying $\mathsf{C}_{1} \to \mathsf{C}_{1} - \mathsf{C}_{2}$ and $\mathsf{C}_{3} \to \mathsf{C}_{3} - {}^{5}\mathsf{C}_{2}$, we get



$$D_{z} = 1000 \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 \\ -1 & 7 & -28 \end{vmatrix}$$
$$= 1000[0 - 1(28 - 0) + 0]$$
$$= 1000 \times (-28)$$
$$= -28000$$
By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-28000}{-16} = 1750$$
$$y = D\frac{y}{D} = \frac{-24000}{-16} = 1500$$
$$z = D\frac{z}{D} = \frac{-28000}{-16} = 1750$$

∴ Amounts of investments are ₹ 1750, ₹ 1500 and ₹ 1750.

Exercise 6.3 | Q 3 | Page 93

Show that the following equations are consistent: 2x + 3y + 4 = 0, x + 2y + 3 = 0, 3x + 4y + 5 = 0

SOLUTION

Given equations are

$$2x + 3y + 4 = 0$$

$$x + 2y + 3 = 0$$

$$3x + 4y + 5 = 0$$

$$\begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix}$$





$$= 2(10 - 12) - 3(5 - 9) + 4(4 - 6)$$

= 2(-2) - 3(-4) + 4(-2)
= -4 + 12 - 8
= 0

... The given equations are consistent.

Exercise 6.3 | Q 4.1 | Page 93

Find k, if the following equations are consistent: x + 3y + 2 = 0, 2x + 4y - k = 0, x - 2y - 3k = 0

SOLUTION

Given equations are

$$x + 3y + 2 = 0$$

 $2x + 4y - k = 0$
 $x - 2y - 3k = 0$

Since, these equations are consistent.

$$\begin{vmatrix} 1 & 3 & 2 \\ 2 & 4 & -k \\ 1 & -2 & -3k \end{vmatrix} = 0$$

$$\therefore 1(-12k - 2k) - 3(-6k + k) + 2(-4 - 4) = 0$$

$$\therefore -14k + 15k - 16 = 0$$

$$\therefore k - 16 = 0$$

$$\therefore k = 16$$

Exercise 6.3 | Q 4.2 | Page 93

Find k, if the following equations are consistent: (k - 2)x + (k - 1)y = 17, (k - 1)x + (k - 2)y = 18, x + y = 5

SOLUTION

Given equations are (k-2)x + (k-1)y = 17 (k-1)x + (k-2)y = 18 x + y = 5Since, these equations are consistent.



$$\therefore \begin{vmatrix} \mathbf{k} - 2 & \mathbf{k} - 1 & -17 \\ \mathbf{k} - 1 & \mathbf{k} - 2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 - R_2$, we get

$$\begin{vmatrix} -1 & 1 & 1 \\ k - 1 & k - 21 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

$$\therefore -1(-5k + 10 + 18) - 1(-5k + 5 + 18) + 1(k - 1 - k + 2) = 0$$

$$\therefore -1 (-5k + 28) - 1(-5k + 23) + 1(1) = 0$$

$$\therefore 5k - 28 + 5k - 23 + 1 = 0$$

$$\therefore 10k - 50 = 0$$

$$\therefore k = 5.$$

Exercise 6.3 | Q 5.1 | Page 93

Find the area of the triangle whose vertices are: (4, 5), (0, 7), (-1, 1)

SOLUTION

Here, A(x₁, y₁) = A(4, 5), B(x₂, y₂) = B(0, 7), C(x₃, y₃) = C(-1, 1)
Area of a triangle =
$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

 $\therefore A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} 4 & 5 & 1 \\ 0 & 7 & 1 \\ -1 & 1 & 1 \end{vmatrix}$
 $= \frac{1}{2}[4(7-1) - 5(0+1) + 1(0+7)]$
 $\therefore A(\Delta ABC) = \frac{1}{2}(24 - 5 + 7)$
= 13 sq.units.



Exercise 6.3 | Q 5.2 | Page 93

Find the area of the triangle whose vertices are: (3, 2), (-1, 5), (-2, -3)

SOLUTION

Here, $A(x_1, y_1) \equiv A(3, 2)$, $B(x_2, y_2) \equiv B(-1, 5)$, $C(x_3, y_3) \equiv C(-2, -3)$

Area of a triangle =
$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

 $\therefore A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ -1 & 5 & 1 \\ -2 & -3 & 1 \end{vmatrix}$
= $\frac{1}{2}[3(5+3) - 2(-1+2) + 1(3+10)]$
= $\frac{1}{2}(24-2+13)$
 $\therefore A(\Delta ABC) = \frac{35}{2}$ sq.unitts.

Exercise 6.3 | Q 5.3 | Page 93

Find the area of the triangle whose vertices are: (0, 5), (0, -5), (5, 0)

SOLUTION

Here, $A(x_1, y_1) \equiv A(0, 5)$, $B(x_2, y_2) \equiv B(0, -5)$, $C(x_3, y_3) \equiv C(5, 0)$

Area of a triangle =
$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

 $\therefore A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} 0 & 5 & 1 \\ 0 & -5 & 1 \\ 5 & 0 & 1 \end{vmatrix}$
= $\frac{1}{2}[0(-5-0) - 5(0-5) + 1(0+25)]$

$$= \frac{1}{2}(0 + 25 + 25)$$
$$= \frac{50}{2}$$

∴ A(Δ ABC) = 25 sq. units

Exercise 6.3 | Q 6 | Page 93

Find the value of k, if the area of the triangle with vertices at A(k, 3), B(-5, 7), C(-1, 4) is 4 square units.

SOLUTION

Here, $A(x_1, y_1) \equiv A(k, 3)$, $B(x_2, y_2) \equiv B(-5, 7)$, $C(x_3, y_3) \equiv C(-1, 4)$ $A(\Delta ABC) = 4$ sq. units

Area of a triangle =
$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

 $\therefore \frac{1}{2} \begin{vmatrix} k & 3 & 1 \\ -5 & 7 & 1 \\ -1 & 4 & 1 \end{vmatrix} = \pm 4$
 $\therefore k(7 - 4) - 3(-5 + 1) + 1(-20 + 7) = \pm 8$
 $\therefore 3k + 12 - 13 = \pm 8$
 $\therefore 3k - 1 = \pm 8$
 $\therefore 3k - 1 = \pm 8$
 $\therefore 3k - 1 = 8 \text{ or } 3k - 1 = -8$
 $\therefore 3k = 9 \text{ or } 3k = -7$
 $\therefore k = 3 \text{ or } k = \frac{-7}{3}$

Exercise 6.3 | Q 7 | Page 93

Find the area of the quadrilateral whose vertices are A(-3, 1), B(-2, -2), C(4, 1), D(2, 3).







SOLUTION

A(-3, 1), B(-2, -2), C(4, 1), D(2, 3)





- \therefore A(\triangle ACD) = 7 sq. units
- $\therefore A(\Delta ABCD) = A(\Delta ABC) + A(\Delta ACD)$

$$= \frac{21}{2} + 7$$
$$= \frac{35}{2}$$
 sq. units

Exercise 6.3 | Q 8 | Page 93

By using determinant, show that the following points are collinear: P(5, 0), Q(10, -3), R(-5, 6)

SOLUTION

Here, $P(x_1, y_1) \equiv P(5, 0), Q(x_2, y_2) \equiv Q(10, -3), R(x_3, y_3) \equiv R(-5, 6)$

If $A(\Delta PQR) = 0$, then the points P, Q, R are collinear.

$$\therefore A(\Delta PQR) = \frac{1}{2} \begin{vmatrix} 5 & 0 & 1 \\ 10 & -3 & 1 \\ -5 & 6 & 1 \end{vmatrix}$$
$$= \frac{1}{2} [5(-3-6) - 0(10+5) + 1(60-15)]$$
$$= \frac{1}{2} (-45+0+45) = 0$$

 $\therefore A(\Delta PQR) = 0$

.: Points P, Q and R are collinear.

Exercise 6.3 | Q 9 | Page 94

The sum of three numbers is 15. If the second number is subtracted from the sum of first and third numbers, then we get 5. When the third number is subtracted from the sum of twice the first number and the second number, we get 4. Find the three numbers.

SOLUTION

Let the three numbers be x, y and z. According to the given conditions, x + y + z = 15x + z - y = 5 i.e. x - y + z = 52x + y - z = 4





$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= 1(1-1) - 1(-1-2) + 1(1+2)$$

$$= 1(0) - 1(-3) + 1(3)$$

$$= 0 + 3 + 3$$

$$= 6 \neq 0$$

$$D_x = \begin{vmatrix} 15 & 1 & 1 \\ 5 & -1 & 1 \\ 4 & 1 & -1 \end{vmatrix}$$

$$= 15(1-1) - 1(-5-4) + 1(5+4)$$

$$= 15(0) - 1(-9) + 1(9)$$

$$= 0 + 9 + 9$$

$$= 18$$

$$D_y = \begin{vmatrix} 1 & 15 & -1 \\ 1 & 5 & 1 \\ 2 & 4 & -1 \end{vmatrix}$$

$$= 1(-5-4) - 15(-1-2) + 1(4-10)$$

$$= 1(-9) - 15(-3) + 1(-6)$$

$$= -9 + 45 - 6$$

$$= 30$$

$$D_z = \begin{vmatrix} 1 & 1 & 15 \\ 1 & -1 & 5 \\ 2 & 1 & 4 \end{vmatrix}$$

$$= 1(-4-5) - 1(4-10) + 15(1+2)$$

$$= 1(-9) - 1(-6) + 15(3)$$

$$= -9 + 6 + 45$$

$$= 42$$

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By Cramer's Rule,

x =
$$\frac{D_x}{D} = \frac{18}{6} = 3$$
,
y = $\frac{D_y}{D} = \frac{30}{6} = 5$,
z = $\frac{D_x}{D} = \frac{42}{6} = 7$

 \therefore The three numbers are 3, 5 and 7.

MISCELLANEOUS EXERCISE 6 [PAGES 94 - 95]Miscellaneous Exercise 6 | Q 1.1 | Page 94Evaluate: $\begin{vmatrix} 2 & -5 & 7 \\ 5 & 2 & 1 \\ 9 & 0 & 2 \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} 2 & -5 & 7 \\ 5 & 2 & 1 \\ 9 & 0 & 2 \end{vmatrix}$$

= $2\begin{vmatrix} 2 & 1 \\ 9 & 2 \end{vmatrix} - (-5)\begin{vmatrix} 5 & 1 \\ 9 & 2 \end{vmatrix} + 7\begin{vmatrix} 5 & 2 \\ 9 & 0 \end{vmatrix}$
= $2(4 - 0) + 5(10 - 9) + 7(0 - 18)$
= $2(4) + 5(1) + 7(-18)$
= $8 + 5 - 126$
= -113 .
Miscellaneous Exercise 6 | Q 1.2 | Page 94

Evaluate:
$$\begin{vmatrix} 1 & -3 & 12 \\ 0 & 2 & -4 \\ 9 & 7 & 2 \end{vmatrix}$$

SOLUTION

$$\begin{vmatrix} 1 & -3 & 12 \\ 0 & 2 & -4 \\ 9 & 7 & 2 \end{vmatrix}$$

= $1\begin{vmatrix} 2 & -4 \\ 7 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 0 & 4 \\ 9 & 2 \end{vmatrix} + 12\begin{vmatrix} 0 & 2 \\ 9 & 7 \end{vmatrix}$
= $1(4 + 28) + 3(0 + 36) + 12(0 - 18)$
= $1(32) + 3(36) + 12(-18)$
= $32 + 108 - 216$
= $-76.$

Miscellaneous Exercise 6 | Q 2.1 | Page 94

Find the value (s) of x, if
$$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & -5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$$

SOLUTION

$$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & -5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$$

$$\therefore 1(-10x^2 + 10x) - 4(5x^2 + 5) + 20(2x + 2) = 0$$

$$\therefore -10x^2 + 10x - 20x^2 - 20 + 40x + 40 = 0$$

$$\therefore -30x^2 + 50x + 20 = 0$$

$$\therefore 3x^2 - 5x - 2 = 0 \quad \dots \text{[Dividding throughtout by (-10)]}$$

$$\therefore 3x^2 - 6x + x - 2 = 0$$

$$\therefore 3x(x - 2) + 1(x - 2) = 0$$

$$\therefore (x - 2) (3x + 1) = 0$$



$$\therefore x - 2 = 0 \text{ or } 3x + 1 = 0$$

$$\therefore x = 2 \text{ or } x = -\frac{1}{3}.$$

Miscellaneous Exercise 6 | Q 2.2 | Page 94

Find the value (s) of x, if $\begin{vmatrix} 1 & 2x & 4x \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0$

SOLUTION

 $\begin{vmatrix} 1 & 2x & 4x \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0$ $\therefore 1(4 - 16) - 2x(1 - 16) + 4x(1 - 4) = 0$ $\therefore 1(-12) - 2x(-15) + 4x(-3) = 0$ $\therefore -12 + 30x - 12x = 0$ $\therefore 18x = 12$ $\therefore x = \frac{12}{18} = \frac{2}{3}.$

Miscellaneous Exercise 6 | Q 3 | Page 95

By using properties of determinants, prove that

$$\begin{vmatrix} x + y & y + z & z + x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

SOLUTION

L.H.S. =
$$\begin{vmatrix} x + y & y + z & z + x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2$, we get

L.H.S. =
$$\begin{vmatrix} x + y + z & z + y + z & x + y + z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Taking (x + y + z) common from R_1 , we get

L.H.S. =
$$(x + y + z) \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

= $(x + y + z) (0)$...[: R_1 and R_3 are identical]
= 0
= R.H.S.

Miscellaneous Exercise 6 | Q 4.1 | Page 95

Without expanding the determinants, show that $\begin{vmatrix} \mathbf{b} + \mathbf{c} & \mathbf{b}\mathbf{c} & \mathbf{b}^2\mathbf{c}^2 \\ \mathbf{c} + \mathbf{a} & \mathbf{c}\mathbf{a} & \mathbf{c}^2\mathbf{a}^2 \\ \mathbf{a} + \mathbf{b} & \mathbf{a}\mathbf{b} & \mathbf{a}^2\mathbf{b}^2 \end{vmatrix} = 0$

SOLUTION

L.H.S. =
$$\begin{vmatrix} \mathbf{b} + \mathbf{c} & \mathbf{bc} & \mathbf{b}^2 \mathbf{c}^2 \\ \mathbf{c} + \mathbf{a} & \mathbf{ca} & \mathbf{c}^2 \mathbf{a}^2 \\ \mathbf{a} + \mathbf{b} & \mathbf{ab} & \mathbf{a}^2 \mathbf{b}^2 \end{vmatrix}$$

Taking bc, ca, ab common from R₁, R₂, R₃ respectively, we get

L.H.S. = (bc)(ca)(ab)
$$\begin{vmatrix} \frac{b-c}{bc} & 1 & bc \\ \frac{c+a}{ca} & 1 & ca \\ \frac{a+b}{ab} & 1 & ab \end{vmatrix}$$

Taking abc common from C_3 , we get



L.H.S. =
$$(a^2b^2c^2)(abc)\begin{vmatrix} \frac{1}{c} + \frac{1}{b} & 1 & \frac{1}{a} \\ \frac{1}{a} + \frac{1}{c} & 1 & \frac{1}{b} \\ \frac{1}{b} + \frac{1}{a} & 1 & \frac{1}{c} \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_3$, we get

L.H.S. =
$$a^{3}b^{3}c^{3}\begin{vmatrix} \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & 1 & \frac{1}{a} \\ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & 1 & \frac{1}{b} \\ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & 1 & \frac{1}{c} \end{vmatrix}$$

Taking
$$\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$
 common from C₁, we get

L.H.S. =
$$a^{3}b^{3}c^{3}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)\begin{vmatrix}1 & 1 & \frac{1}{a}\\1 & 1 & \frac{1}{b}\\1 & 1 & \frac{1}{c}\end{vmatrix}$$

$$= a^{3}b^{3}c^{3}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)(0) \dots [C_{1} \text{ and } C_{2} \text{ are identical}]$$

= R.H.S.

Miscellaneous Exercise 6 | Q 4.2 | Page 95

Without expanding the determinants, show that $\begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

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SOLUTION

L.H.S. =
$$\begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$$

= Taking a, b, c common from C1, C2, C3 respectively, we get

L.H.S. = abc
$$\begin{vmatrix} x & y & z \\ a & b & c \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix}$$

= $\begin{vmatrix} x & y & z \\ a & b & c \\ \frac{abc}{a} & \frac{abc}{b} & \frac{abc}{c} \end{vmatrix}$
= $\begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

Miscellaneous Exercise 6 | Q 4.3 | Page 95

	l	\mathbf{m}	\mathbf{n}		\mathbf{n}	f	w	
Without expanding the determinants, show that	е	d	f	=	l	e	u	
	\mathbf{u}	v	w		m	\mathbf{d}	v	

SOLUTION

$$L.H.S. = \begin{vmatrix} l & m & n \\ e & d & f \\ u & v & w \end{vmatrix}$$

Interchanging rows and columns, we get

L.H.S. =
$$\begin{vmatrix} l & e & u \\ m & d & v \\ n & f & w \end{vmatrix}$$

Applying $R_2 \leftrightarrow R_3$, we get

L.H.S. = $\begin{vmatrix} l & e & u \\ m & f & w \\ m & d & v \end{vmatrix}$ Applying $R_1 \leftrightarrow R_2$, we get L.H.S. = $\begin{vmatrix} n & f & w \\ l & e & u \\ m & d & v \end{vmatrix}$ = R.H.S.

Without expanding the determinants, show that $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$

SOLUTION

$$Let D = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

Taking (- 1) common from R₁, R₂, R₃, we get

$$D = (-1)^{3} \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Interchanging rows and columns, we get

$$D = -1 \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$
$$\therefore D = -1(D)$$

$$\therefore 2D = 0$$

$$\therefore D = 0$$

$$\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0.$$

Miscellaneous Exercise 6 | Q 5.1 | Page 95

Solve the following linear equations by Cramer's Rule: 2x - y + z = 1, x + 2y + 3z = 8, 3x + y - 4z = 1

SOLUTION

Given equations are 2x - y + z = 1x + 2y + 3z = 83x + y - 4z = 1 $D = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{vmatrix}$ = 2(-8-3) - (-1)(-4-9) + 1(1-6)= 2(-11) + 1(-13) + 1(-5)= -22 - 13 - 5 $= -40 \neq 0$ $D_{X} = \begin{vmatrix} 1 & -1 & 1 \\ 8 & 2 & 3 \\ 1 & 1 & -4 \end{vmatrix}$ = 1(-8-3) - (-1)(-32 - 30 + 1(8 - 2))= 1(-11) + 1(-35) + 1(6)= - 11 - 35 + 6 = -40



$$D_{y} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 8 & 3 \\ 3 & 1 & -4 \end{vmatrix}$$

= 2(-32-3) - 1(-4-9) + 1(1-24)
= 2(-35) - 1(-13) + 1(-23)
= -70 + 13 - 23
= -80
$$D_{z} = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 8 \\ 3 & 1 & 1 \end{vmatrix}$$

= 2(2-8) - (-1)(1-24) + 1(1-6)
= 2(-6) + 1(-23) + 1(-5)
= -12 - 23 - 5
= -40

By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-40}{-40} = 1$$
$$y = \frac{D_y}{D} = \frac{-80}{-40} = 2,$$
$$z = \frac{D_z}{D} = \frac{-40}{-40} = 1$$

 \therefore x = 1, y = 2 and z = 1 are the solutions of the given equations.

Miscellaneous Exercise 6 | Q 5.2 | Page 95

Solve the following equations using Cramer's Rule:

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -2, \ \frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 3, \ \frac{2}{x} - \frac{1}{y} + \frac{3}{z} = -1$$
SOLUTION

Let $\frac{1}{x}$ p, $\frac{1}{y}$ = q, $\frac{1}{z}$ = r .: The given equations become p + q + r = -2p - 2q + r = 32p - q + 3r = -1 $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 2 & -1 & 3 \end{vmatrix}$ = 1(-6 + 1) - 1(3 - 2) + 1(-1 + 4)= 5 - 1 + 3= - 3 $\mathsf{D}_{\mathsf{p}} = \begin{vmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & -1 & 3 \end{vmatrix}$ = -2(-6 + 1) - 1(9 + 1) + 1(-3 - 2)= 10 - 10 - 5 = - 5 $D_{q} = \begin{vmatrix} 1 & -2 & 1 \\ 1 & 3 & 1 \\ 2 & -1 & 3 \end{vmatrix}$ = 1(9 + 1) + 2(3 - 2) + 1(-1 - 6)= 10 + 2 - 7= 5 $\mathsf{D}_{\mathsf{r}} = \begin{vmatrix} 1 & 1 & -2 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}$

$$= 1(2 + 3) - 1(-1 - 6) - 2(-1 + 4)$$
$$= 5 + 7 - 6$$
$$= 6$$

By Cramer's Rule,

$$p = \frac{D_p}{D} = \frac{-5}{-3} = \frac{5}{3}$$

$$q = D - \frac{q}{q} = \frac{-5}{3},$$

$$r = \frac{D_r}{D} = \frac{6}{-3} = -2$$

$$\therefore \frac{1}{x} = \frac{5}{3}, \frac{1}{y} = \frac{-5}{3}, \frac{1}{z} = -2$$

$$\therefore x = \frac{3}{5}, y = \frac{-3}{5}, z = \frac{-1}{2}$$
are the solution of the given equatios.

Miscellaneous Exercise 6 | Q 5.3 | Page 95

Solve the following linear equations by Cramer's Rule: x - y + 2z = 7, 3x + 4y - 5z = 5, 2x - y + 3z = 12

SOLUTION

Given equations are

$$\begin{aligned} x - y + 2z &= 7\\ 3x + 4y - 5z &= 5\\ 2x - y + 3z &= 12\\ D &= \begin{vmatrix} 1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3 \end{vmatrix} \\ &= 1(12 - 5) - (-1)(9 + 1) + (-3 - 8)\\ &= 1(7) + 1(19) + 2(-11) \end{aligned}$$

$$= 7 + 19 - 22$$

= $4 \neq 0$
$$D_{x} = \begin{vmatrix} 7 & -1 & 2 \\ 5 & 4 & -5 \\ 12 & -1 & 3 \end{vmatrix}$$

= $7(12 \neq 5) - (-1)(15 + 60) + 2(-5 - 48)$
= $7(7) + 1(75) + 2(-53)$
= $49 + 75 - 106$
= 18
$$D_{y} = \begin{vmatrix} 1 & 7 & 2 \\ 3 & 5 & -5 \\ 2 & 12 & 3 \end{vmatrix}$$

= $1(15 + 60) - 7(9 + 10) + 2(36 - 10)$
= $1(75) - 7(9) + 2(26)$
= $75 - 133 + 52$
= -6
$$D_{z} = \begin{vmatrix} 1 & -1 & 7 \\ 3 & 4 & 5 \\ 2 & -1 & 12 \end{vmatrix}$$

= $1(48 + 5) - (-1)(36 - 1) + 7(-3 - 8)$
= $1(53) (26) 7(-1)$
= $3 + 26 - 77$
= 2
By Cramer's Rule,
 $x = \frac{D_{x}}{D} = \frac{18}{4} = \frac{9}{2},$
 $y = \frac{D_{y}}{D} = \frac{-3}{4} = \frac{-3}{2},$

$$z = \frac{D_z}{D} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore x = \frac{9}{2}, y = \frac{-3}{2} \text{ and } z = \frac{1}{2} \text{ are the solutions of the given equations.}$$

Miscellaneous Exercise 6 | Q 6.1 | Page 95

Find the value (s) of k, if the following equations are consistent: 3x + y - 2 = 0, kx + 2y - 3 = 0 and 2x - y = 3

SOLUTION

Given equations are

3x + y - 2 = 0 kx + 2y - 3 = 0 2x - y = 3 i.e. 2x - y - 3 = 0Since, these equations are consistent. $\begin{vmatrix}3 & 1 & -2 \\ k & 2 & -3 \\ 2 & -1 & -3\end{vmatrix} = 0$ $\therefore 3(-6 - 3) - 1(-3k + 6) - 2(-k - 4) = 0$ $\therefore 3(-9) - 1(-3k + 6) - 2(-k - 4) = 0$ $\therefore 3(-9) - 1(-3k + 6) - 2(-k - 4) = 0$ $\therefore -27 + 3k - 6 + 2k + 8 = 0$ $\therefore 5k - 25 = 0$ $\therefore k = \frac{25}{5}$ = 5.

Miscellaneous Exercise 6 | Q 6.2 | Page 95

Find the value (s) of k, if the following equations are consistent: kx + 3y + 4 = 0, x + ky + 3 = 0, 3x + 4y + 5 = 0

SOLUTION

Given equations are kx + 3y + 4 = 0x + ky + 3 = 0





3x + 4y + 5 = 0Since, these equations are consistent.

$$\begin{vmatrix} k & 3 & 4 \\ 1 & k & 3 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$$\therefore k(5k - 12) - 3(5 - 9) + 4(4 - 3k) = 0$$

$$\therefore 5k^{2} - 12k + 12 + 16 - 12k = 0$$

$$\therefore 5k^{2} - 24k + 28 = 0$$

$$\therefore 5k^{2} - 24k + 28 = 0$$

$$\therefore 5k^{2} - 10k - 14k + 28 = 0$$

$$\therefore 5k(k - 2) - 14(k - 2) = 0$$

$$\therefore (k - 2) (5k - 14) = 0$$

$$\therefore k - 2 = 0 \text{ or } 5k - 14 = 0$$

$$\therefore k = 2 \text{ or } k = \frac{14}{5}$$

Miscellaneous Exercise 6 | Q 7.1 | Page 95

Find the area of triangles whose vertices are A(-1, 2), B(2, 4), C(0, 0)

SOLUTION

Here, $A(x_1, y_1) \equiv A(-1, 2)$, $B(x_2, y_2) \equiv B(2, 4)$, $C(x_3, y_3) \equiv C(0, 0)$

Area of a triangle =
$$\frac{1}{2} \begin{vmatrix} x1 & y1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

 $\therefore A(\Delta ABC) = \frac{1}{2} \begin{vmatrix} -1 & 2 & 1 \\ 2 & 4 & 1 \\ 0 & 0 & 1 \end{vmatrix}$
= $\frac{1}{2} [-1(4-0) - 2(2-0) + 1(0-0)]$
= $\frac{1}{2} (-4-4)$

$$=\frac{1}{2}(-8)$$

Since, area cannot be negative.

 \therefore A(\triangle ABC) = 4 sq. units

Miscellaneous Exercise 6 | Q 7.2 | Page 95 Find the area of triangles whose vertices are P(3, 6), Q(-1, 3), R(2, -1)

SOLUTION

Here, $P(x_1, y_1) \equiv P(3, 6)$, $Q(x_2, y_2) \equiv Q(-1, 3)$, $R(x_3, y_3) \square R(2, -1)$

Area of triangle =
$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

 $\therefore A(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 3 & 6 & 1 \\ -1 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix}$
= $\frac{1}{2}[3(3+1) - 6(-1-2) + 1(1-6)]$
= $\frac{1}{2}[3(4) - 6(-3) + 1(-5)]$
= $\frac{1}{2}(12 + 18 - 5)$
 $\therefore A(\Delta PQR) = \frac{25}{2}$ sq. units

Miscellaneous Exercise 6 | Q 7.3 | Page 95

Find the area of triangles whose vertices are L(1, 1), M(-2, 2), N(5, 4)

SOLUTION

Here, $L(x_1, y_1) \equiv L(1, 1)$, $M(x_2, y_2) \equiv M(-2, 2)$, $N(x_3, y_3) \equiv N(5, 4)$



Area of a triangle =
$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

 $\therefore A(\Delta LMN) = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -2 & 2 & 1 \\ 5 & 4 & 1 \end{vmatrix}$
= $\frac{1}{2}[1(2-4) - 1(-2-5) + 1(-8-10)]$
= $\frac{1}{2}[1(-2) - 1(-7) + 1(-18)]$
= $\frac{1}{2}(-2+7-18)$
= $-\frac{13}{2}$

Since, area cannot be negative.

 \therefore A(Δ LMN) = $\frac{13}{2}$ sq. units

Miscellaneous Exercise 6 | Q 8.1 | Page 95

Find the value of k, if area of $\triangle PQR$ is 4 square units and vertices are P(k, 0), Q(4, 0), R(0, 2).

SOLUTION

Here, $P(x_1, y_1) \equiv P(k, 0)$, $Q(x_2, y_2) \equiv Q(4, 0)$, $R(x_3, y_3) \equiv R(0, 2)$ $A(\Delta PQR) = 4$ sq. units

Area of a triangle =
$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

 $\therefore \pm 4 = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$$\therefore \pm 4 = \frac{1}{2} [k(0-2) - 0 + 1(8-0)]$$

$$\therefore \pm 8 = -2k + 8$$

$$\therefore 8 = -2k + 8 \text{ or } -8 = -2k + 8$$

$$\therefore -2k = 0 \qquad \text{or } 2k = 16$$

$$\therefore k = 0 \qquad \text{or } k = 8$$

Miscellaneous Exercise 6 | Q 8.2 | Page 95

Find the value of k, if area of Δ LMN is 33/2 square units and vertices are L(3, - 5), M(- 2, k), N(1, 4).

SOLUTION

Here,
$$L(x_1, y_1) \equiv L(3, -5)$$
, $M(x_2, y_2) \equiv M(-2, k)$, $N(x_3, y_3) \equiv N(1, 4)$
 $A(\Delta LMN) = \frac{33}{2}$ q. units
Area of a triangle $= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$
 $\therefore \pm \frac{33}{2} = \frac{1}{2} \begin{vmatrix} 3 & -5 & 1 \\ -2 & k & 1 \\ 1 & 4 & 1 \end{vmatrix}$
 $\therefore \pm \frac{33}{2} = \frac{1}{2} [3(k-4) - (-5)(-2-1) + 1(-8-k)]$
 $\therefore \pm 33 = 3k - 12 - 5 - 8 - k$
 $\therefore \pm 33 = 2k - 35$
 $\therefore 2k - 35 = 33$ or $2k - 35 = -33$
 $\therefore 2k = 68$ or $2k = 2$
 $\therefore k = 34$ or $k = 1$

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